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Constraining the Power Spectrum Using the Column Density Distribution: a Status Report

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ABSTRACT

We review the arguments given in [1] for how the slope of the column density distribution of the Lyα forest should depend on the matter power spectrum. The latest progress, presented by various groups in this conference and elsewhere, is summarized.

1.   Introduction

One of the results coming out of the first hydrodynamic simulations of the low column density Lyα forest ($N_{HI} \lesssim 10^{14.5}$ cm^{-2}) is that the predicted column density distribution (we will denote by $f(N_{HI})$) roughly agrees with observations ([2],[3], [4] & [5]). Because of the limited number of cosmological models simulated then, it was unclear if the agreement is coincidental or a generic prediction of a large class of models. Subsequent analytic/semi-analytic work using the lognormal approximation for the density field ([6] & [7]) predicts that $f(N_{HI})$'s for a large class of models are very similar. It is later shown by [1] that 1. the lognormal approximation does not predict $f(N_{HI})$ accurately when compared with the result of a hydrodynamic simulation, although it provides useful physical insights; 2. the Zel'dovich approximation (ZA hereafter), which has been widely tested and used in studies of large scale structure [8], works better. In addition, it is pointed out that the slope of $f(N_{HI})$ depends on both the normalization and slope of the power spectrum, and, using the ZA, it is predicted that cosmological models with less small scale power would have a steeper $f(N_{HI})$.

We review arguments that lead to this conclusion, discuss the possible pitfalls, and summarize the latest progress.

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2.   Less Power Implies Steeper $f(N_{HI})$

One starts with the following picture of the forest: each Lyα line is associated with a local maximum or a peak in the neutral hydrogen density. The column density for a given peak is:

$$N_{HI} = \int_{peak} n_{HI} dr = [n_{HI}L]_{peak} \qquad (1)$$

where r is the proper distance along the line of sight, n_{HI} is the proper density of HI, and the symbol $[\,]_{peak}$ denotes evaluation at the peak. L is the width of the peak defined by $L \equiv \sqrt{-2\pi/[d^2 \ln n_{HI}/dr^2]}$. The above equation can be derived by replacing n_{HI} under the integral by $\exp[\ln n_{HI}]$ and performing a Gaussian integration after Taylor expanding $\ln n_{HI}$ to second order.

n_{HI} is proportional to the underlying baryon or gas density ρ raised to some power, which depends on the reionization history [9]. N_{HI} is then a simple function of the peak height in ρ and its second derivative $d^2\rho/dr^2$ which determines the peak width. Hence, $f(N_{HI})$, which is the number of absorption lines per unit N_{HI} per unit redshift, is a statistic of density peaks: the number density of peaks with a given combination of peak height and width.

Such a statistic can be computed using the method of [10]. It turns out that for the low column density Lyα forest ($N_{HI} \lesssim 10^{14.5}$ cm^{-2}), the relevant overdensity $\delta\rho/\bar\rho$ is of the order of unity. The ZA is known to work well for such a quasilinear density field, and it can be shown generally that the Gaussian nature of the ZA displacement field implies the slope of $f(N_{HI})$ depends on the normalization and slope of the primordial power spectrum [1]. We will not discuss the dependence on the power spectrum slope in detail here, because its effect is smaller than that of the normalization, and also because the arguments leading to it are more intricate. The interested reader is referred to [1].

The effect of the power spectrum normalization is simple to understand: let us compare two models, A and B, with A having more power on the relevant scales (see next sec.) than B. **Model B, because it has less power, and so has a less nonlinear density field, would have proportionally fewer high peaks compared to low peaks.** Now, N_{HI} depends on both the peak height and width (eq. 1), but it turns out that the peak width is correlated with peak height in such a way that a larger peak height also means higher N_{HI}. Therefore, B, having fewer high peaks compared to low peaks, also has fewer high N_{HI} lines compared to low ones, hence a steeper $f(N_{HI})$. We illustrate this in Fig. 1 where the CDM model plays the role of A and the CHDM model that of B. Also shown in the plot is a comparison of the ZA + peak-counting prediction with the result of hydrodynamic-simulation + Voigt-profile-fitting for the CDM model. The agreement is encouraging.

3. Discussion

Let us examine a few caveats to the above arguments, and discuss some recent work which has bearing on these issues. First, the peak-counting is done in real space, whereas the absorption lines are observed in velocity space. A plausible support for our procedure comes from a test in [1] where $f(N_{\mathrm{HI}})$'s are computed for two sets of spectra, one generated with non-zero peculiar velocities, and the other with vanishing ones. They agree with each other well. Moreover, one can see from eq. (1) that for an isolated peak, the velocity structure plays no role in determining N_{HI}. It is interesting to note that there is a recent paper in which peak-counting is done directly in velocity space [11].

Second, when using the ZA, smoothing has to be applied to the initial displacement field to minimize the amount of orbit-crossing by the time of interest. There is a well-tested prescription for the optimal dark matter smoothing scale, which we call the orbit-crossing scale [8]. Another relevant smoothing scale is roughly the Jeans length, which specifies the scale on which the baryon density is smoothed with respect to the dark matter density [12]. It turns out that both are of the order of $k^{-1} \sim 0.1h^{-1}\mathrm{Mpc}$, which makes the Ly$\alpha$ forest interesting, because it constrains the power spectrum on these scales which are smaller than those probed by galaxy surveys. The procedure adopted in [1] is to smooth the initial displacement field on either the orbit-crossing or the Jeans scale, whichever is larger. However, while the Jeans-smoothing is physically motivated in the sense it is meant to model the effect of finite gas pressure, the other smoothing is more of a corrective measure to deal with the inaccuracy of the ZA after orbit-crossing. It is a legitimate concern whether such smoothing erases structures that might contribute significantly to $f(N_{\mathrm{HI}})$. The test against a hydrodynamic simulation shown in Fig. 1 lends support to the use of the smoothed ZA. However, this might be a lucky coincidence. Further tests are needed.

Third, there is no guarantee that in current structure formation models, a given peak in optical depth has exactly the Voigt-profile shape, and so standard profile-fitting algorithms might result in one single peak being fitted by several small profiles, which means the association of the one single peak with one absorption line is not exact. The average effect on $f(N_{\mathrm{HI}})$ is hard to predict analytically, and has to be checked through simulations. Let us now turn to the latest work that has bearing on the above issues.

In two separate pieces of work presented in this conference ([13] & [14]), it is shown using new hydrodynamic simulations that the slope of $f(\mathrm{N_{HI}})$ is roughly the same for several cosmological models (of the order of 4 for each). This might indicate the third worry mentioned above is justified: that the nature of the Voigt-profile fitting procedure might conspire to result in the same $f(\mathrm{N_{HI}})$ for different models, contrary to what would be expected using a peak-counting method.

However, it could also be the case that the models simulated above do not span a large enough range (or have small enough small-scale power) to allow one to see the effect on $f(N_{HI})$. A different line of attack is developed in [12], in which the Lyα forest is simulated using a PM code, with the Poisson solver modified to compute an effective potential due to pressure in addition to that due to gravity. A related approach is advocated by [15]. This method produces results in good agreement with hydrodynamic simulations for the low N_{HI} Lyα forest. It avoids the problem of the uncertain smoothing scale for the ZA. It also allows simulations of a large number of models with relatively modest computer expense. This is undertaken by [16], who finds appreciable differences in the slope of $f(N_{HI})$ among a set of 25 cosmological models, and concludes the normalization of the power spectrum at a particular scale can indeed be constrained by $f(N_{HI})$, confirming the prediction of [1]. It is also shown that the peak-counting method in real space compares favorably with Voigt-profile-fitting as a way of finding $f(N_{HI})$. However, a few caveats have to be kept in mind. First, in [16], only a narrow range of N_{HI} ($10^{13}-10^{14}\,\mathrm{cm^{-2}}$) is used to arrive at the above conclusions. Second, the correction introduced due to the finite box size should be checked using larger simulations. Third, the observed $f(N_{HI})$ that the theoretical predictions are compared against is obtained using a different profile-fitting algorithm from the one discussed in [16]. In general, the proper error-analysis is probably highly algorithm-dependent, when one attempts to use the observed $f(N_{HI})$ to constrain the power spectrum.

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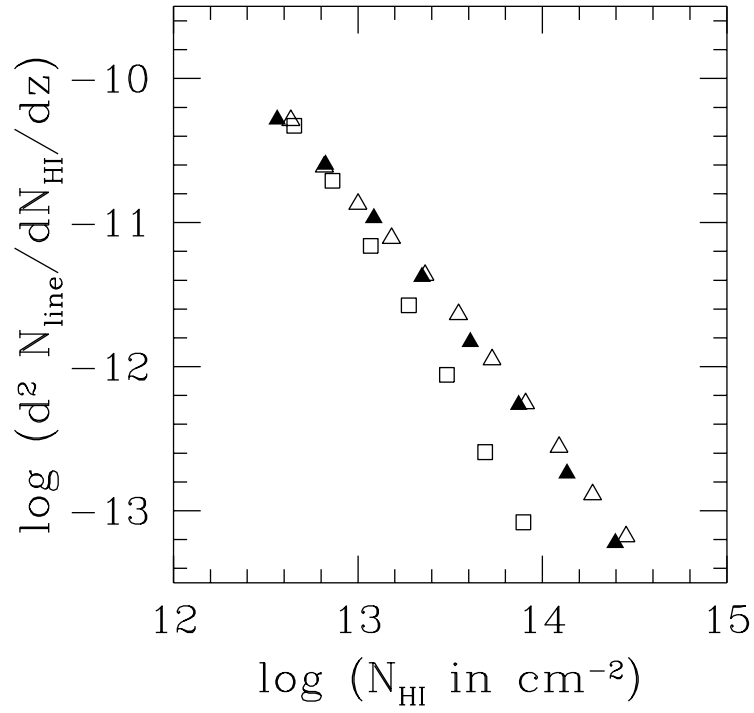
Fig. 1.— The column density distributions for a CDM model (*triangle*) and a CHDM model (*square*). Open/filled symbols are obtained using the ZA/hydrodynamic simulation respectively. See [17] & [1] for details.